

# CH 3 TRIGONOMETRY

## ANSWERS AND EXPLANATIONS

### EXERCISE 1

1. (a)  $\pi$  radians =  $180^\circ$

Therefore,  $\frac{3\pi}{5}$  radians =  $\frac{3 \times 180^\circ}{5} = 108^\circ$

2. (d) 
$$\frac{7 \sin A - 3 \cos A}{7 \sin A + 2 \cos A} = \frac{7 \frac{\sin A}{\cos A} - 3}{7 \frac{\sin A}{\cos A} + 2}$$

(Dividing num. & denom. by  $\cos A$ )

$$= \frac{7 \times \frac{4}{7} - 3}{7 \times \frac{4}{7} + 2} \quad \left[ \frac{\sin A}{\cos A} = \frac{4}{7} \right]$$

$$= \frac{4 - 3}{4 + 2} = \frac{1}{6}$$

3. (c) For  $0 < x \leq \pi/2$

$\sin x$  and  $\operatorname{cosec} x$  both are positive

We know that *AM* of two positive numbers  $\geq$  their *GM*.

Thus,  $\frac{\sin x + \operatorname{cosec} x}{2} \geq \sqrt{\sin x \cdot \operatorname{cosec} x}$

or  $\sin x + \operatorname{cosec} x \geq 2$

Thus, the least value is 2.

4. (b)  $7 \operatorname{cosec} \theta - 3 \cot \theta = 7$

$7 \operatorname{cosec} \theta = (7 + 3 \cot \theta)$

or  $49 \operatorname{cosec}^2 \theta = 49 + 9 \cot^2 \theta + 42 \cot \theta$

or  $49(1 + \cot^2 \theta) = 49 + 9 \cot^2 \theta + 42 \cot \theta$

or  $40 \cot^2 \theta = 42 \cot \theta$  or

$\cot \theta = \frac{21}{20}$

Now,  $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta = 1 + \frac{441}{400} = \frac{841}{400}$

or  $\operatorname{cosec} \theta = \frac{29}{20}$

Therefore,  $7 \cot \theta - 3 \operatorname{cosec} \theta = 7 \times \frac{21}{20} - 3 \times \frac{29}{20}$

$$= \frac{147}{20} - \frac{87}{20} = \frac{60}{20} = 3$$

5. (b)  $\sin \theta + \cos \theta = \sqrt{2} \left( \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta \right)$

$$= \sqrt{2} \left( \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} \right) = \sqrt{2} \sin \left( \theta + \frac{\pi}{4} \right)$$

Max. value of  $\sin \left( \theta + \frac{\pi}{4} \right) = 1$

when  $\theta + \frac{\pi}{4} = \frac{\pi}{2}$  or  $\theta = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

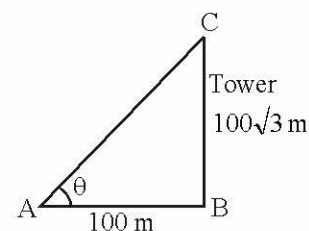
Hence,  $\sin \theta + \cos \theta$  is max. at  $\theta = \pi/4$

6. (d)  $\cos^6 x + \sin^6 x + 3 \sin^2 x \cos^2 x$

$$= (\cos^2 x)^3 + (\sin^2 x)^3 + 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$$

$$= (\cos^2 x + \sin^2 x)^3 = 1$$

7. (c)

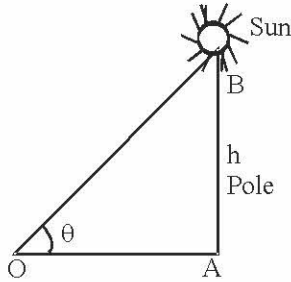


$\theta$  is the angle of elevation of the top of the tower (C).

$$\therefore \tan \theta = \frac{100\sqrt{3}}{100} = \sqrt{3}$$

$$\Rightarrow \theta = 60$$

8. (b) In rt.  $\Delta OAB$ ,  $\frac{AB}{OA} = \tan \theta$



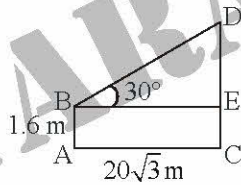
$$\frac{h}{h} = \tan \theta$$

$$(\because OA = AB = h)$$

$$\Rightarrow \tan \theta = 1 \Rightarrow \theta = 45^\circ.$$

9. (a) Let AB be the observer and CD be the tower.

Draw  $BE \perp CD$ .

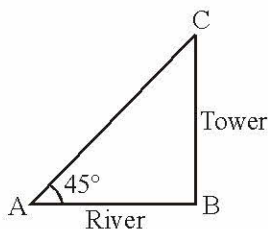


Then,  $CE = AB = 1.6$  m,  $BE = AC = 20\sqrt{3}$  m.

$$\frac{DE}{BE} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}} \text{ m} = 20 \text{ m.}$$

$$\therefore CD = CE + DE = (1.6 + 20) \text{ m} = 21.6 \text{ m.}$$

10. (c)

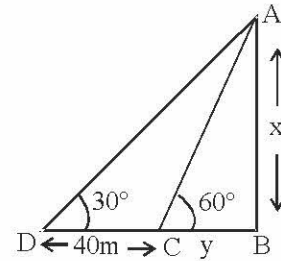


AB is the river and BC is the tower.

$$\therefore \frac{BC}{AB} = \tan 45^\circ = 1 \Rightarrow BC = AB$$

11. (d) Let height of tower  $AB = x$  m and  $BC = y$  m,  $DC = 40$  m.

In  $\Delta ABC$ ,



$$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{x}{y} = \sqrt{3} \Rightarrow y = \frac{x}{\sqrt{3}}$$

...(i)

Now In rt  $\Delta ABD$ ,  $\frac{AB}{BD} = \tan 30^\circ$

$$\Rightarrow \frac{x}{40+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}x = 40+y \Rightarrow \sqrt{3}x = 40 + \frac{x}{\sqrt{3}} \quad \text{[using (i)]}$$

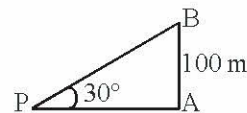
(i)

$$\Rightarrow 3x = 40\sqrt{3} + x \Rightarrow 3x - x = 40\sqrt{3}$$

$$\Rightarrow 2x = 40\sqrt{3}$$

$$x = 20\sqrt{3} \text{ m}; \therefore y = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m.}$$

12. (c) Let B be the tower. Then,  $\angle APB = 30^\circ$  and  $AB = 100$  m.



$$\frac{AB}{AP} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow (AB \times \sqrt{3}) = 100\sqrt{3} \text{ m.}$$



$$= (100 \times 1.73) \text{ m}$$

$$= 173 \text{ m.}$$

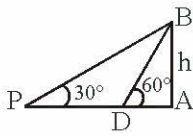
18. (3)  $\cot 10^\circ \cdot \cot 80^\circ \cdot \cot 20^\circ \cdot \cot 70^\circ \cdot \cot 60^\circ$

$$= \cot 10^\circ \cdot \tan 10^\circ \cdot \cot 20^\circ \cdot \tan 20^\circ \cdot \cot 60^\circ$$

$$\left[ \begin{aligned} \because \tan(90^\circ - \theta) &= \cot \theta \\ \tan \theta \cdot \cot \theta &= 1 \end{aligned} \right]$$

$$= 1 \cdot 1 \cdot \sqrt{3} = \sqrt{3}$$

13. (d)



One of AB, AD and CD must have been given.

So, the data is inadequate.

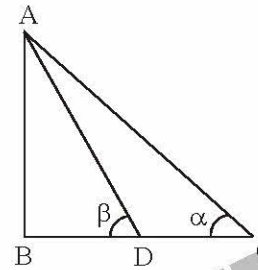
19. (3)

14. (a) Let h be the height.

$$\text{Then, } 6 : 4 = h : 50$$

$$\Rightarrow h = \frac{50 \times 6}{4} = 75 \text{ m}$$

15. (c) The broken part will become the hypotenuse and the remaining part will be perpendicular so angle opposite to it become  $30^\circ$ . Since side opposite to  $30^\circ$  angle is half of the hypotenuse.



AB = monument = h metre

DC = 138 metre

BD = x metre

$$\tan \alpha = \frac{1}{5}$$

$$\sec \beta = \frac{\sqrt{193}}{12}$$

$$\therefore \tan \beta = \sqrt{\sec^2 \beta - 1}$$

$$= \sqrt{\frac{193}{144} - 1} = \sqrt{\frac{193 - 144}{144}}$$

$$= \frac{\sqrt{49}}{\sqrt{144}} = \frac{7}{12}$$

$\therefore$  From  $\Delta ABC$ ,

$$\tan \alpha = \frac{AB}{BC} \Rightarrow \frac{1}{5} = \frac{h}{x + 138}$$

$$\Rightarrow h = \frac{x + 138}{5}$$

$$\Rightarrow 5h = x + 138$$

...(i)

From  $\Delta ABD$ ,

16. (2)  $\sin \alpha + \cos \beta = 2$

$$\sin \alpha \leq 1; \cos \beta \leq 1$$

$$\Rightarrow \alpha = 90^\circ; \beta = 0^\circ$$

$$\therefore \sin\left(\frac{2\alpha + \beta}{3}\right) = \sin\left(\frac{180^\circ}{3}\right)$$

$$= \sin 60^\circ = \frac{\sqrt{3}}{2}$$

17. (3)  $\cos^4 \theta - \sin^4 \theta = \frac{2}{3}$

$$\Rightarrow (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)$$

$$= \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - (1 - \cos^2 \theta) = \frac{2}{3}$$

$$\Rightarrow 2\cos^2 \theta - 1 = \frac{2}{3}$$



$$\tan \beta \frac{h}{x} \Rightarrow \frac{7}{12} = \frac{h}{x}$$

$$\Rightarrow x = \frac{12h}{7} \quad \dots(ii)$$

$$\therefore 5h = \frac{12h}{7} = 138$$

$$\Rightarrow 35h - 12h = 138 \times 7$$

$$\Rightarrow 23h = 138 \times 7$$

20. (1)  $\frac{\sin \alpha}{\cos(30^\circ + \alpha)} = 1$

$$\Rightarrow \frac{\sin \alpha}{\sin(90^\circ - 30^\circ - \alpha)} = 1$$

$$\Rightarrow \frac{\sin \alpha}{\sin(60^\circ - \alpha)} = 1$$

$$\Rightarrow \sin \alpha = \sin(160^\circ - \alpha)$$

$$\Rightarrow \alpha = 60^\circ - \alpha$$

$$\Rightarrow 2\alpha = 60^\circ \Rightarrow \alpha = 30^\circ$$

$$\therefore \sin \alpha + \cos 2\alpha$$

$$= \sin 30^\circ + \cos 60^\circ$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

21. (1)  $\tan \theta = 1 \Rightarrow \theta = 45^\circ$

$$\therefore \frac{8 \sin \theta + 5 \cos \theta}{\sin^3 \theta - 2 \cos^3 \theta + 7 \cos \theta}$$

$$= \frac{8 \times \frac{1}{\sqrt{2}} + \frac{5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}}$$

$$= \frac{\frac{13}{\sqrt{2}}}{\frac{13}{2\sqrt{2}}} = \frac{13}{\sqrt{2}} \times \frac{2\sqrt{2}}{13} = 2$$

22. (1)  $\cos^2 \theta + \cos^4 \theta = 1$

$$\Rightarrow \cos^4 \theta = 1 - \cos^2 \theta = \sin^2 \theta$$

$$\Rightarrow \tan^2 \theta = \cos^2 \theta$$

$$\therefore \tan^2 \theta + \tan^4 \theta = \cos^2 \theta + \cos^4 \theta = 1$$

23. (2)  $\tan 4^\circ \cdot \tan 43^\circ \cdot \tan 47^\circ \cdot \tan 86^\circ$

$$= (\tan 4^\circ \cdot \tan 86^\circ) (\tan 43^\circ \cdot \tan 47^\circ)$$

$$= (\tan 4^\circ \cdot \cot 4^\circ) (\tan 43^\circ \cdot \cot 43^\circ)$$

$$= 1$$

$$\left[ \begin{array}{l} \tan(90^\circ - \theta) = \cot \theta; \\ \tan \theta \cdot \cot \theta = 1 \end{array} \right]$$

24. (3)  $\tan \theta = \frac{4}{3}$  (Given)

$$\therefore \frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta} = \frac{3 \tan \theta + 2}{3 \tan \theta - 2}$$

$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{4 + 2}{4 - 2} = 3$$

25. (4)  $\sec 17^\circ - \sin 73^\circ$

$$= \sec 17^\circ - \sin(90^\circ - 17^\circ)$$

$$= \sec 17^\circ - \cos 17^\circ$$

$$= \frac{1}{\cos 17^\circ} - \cos 17^\circ$$

$$= \frac{1 - \cos^2 17^\circ}{\cos 17^\circ} = \frac{\sin^2 17^\circ}{\cos 17^\circ}$$

$$= \frac{\frac{x^2}{y^2}}{\sqrt{1 - \frac{x^2}{y^2}}}$$



